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# Mode Spectrum of Chiral Resonators

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## Abstract

The mode spectrum of a circular waveguide resonator filled with chiral material is calculated. For verification purposes the model is fed with chiral material parameters extracted from standard waveguide measurements. The resonance frequencies computed for different lengths of the resonator are in good agreement with those obtained from an experimental resonator setup.

## 1. Introduction

Cavity resonators are in general well suited for high precision material characterization. This is because their resonant frequency and quality factor depend sensitively on the electromagnetic properties of the enclosed material. To extract the constitutive parameters of chiral media from such experiments, however, is a complicated and demanding task, both experimentally and theoretically [1]. An alternative use of the resonator is therefore proposed here. Considering a completely filled cavity the material parameters cannot be extracted because the measurement does not provide enough information. The measured resonator characteristics, however, can still be used to accurately verify the material parameters obtained from other experiments. This has to be done indirectly by comparing the measured and the computed responses of the resonator. For an accurate check a field theoretical description of the resonator must be used. This model is fed with the data to be verified.

A full-wave theory of circular waveguide resonators filled with chiral material was developed in [2]. In this contribution the model is extended to cover the whole mode spectrum, in particular the modes with no length dependence. Next, the experimental setup is described and, finally, both the computed and the measured resonant frequencies are compared for different lengths of the resonator.

## 2. Theory

The resonator consists of a lossless circular waveguide of length  $l$  and radius  $a$ . It is short-circuited at both ends and filled with chiral material. As was explained in [2] the non-degeneracy of left and right circulating modes in circular chirowaveguides calls for a mode expansion at the short-circuited ends of the resonator. Since modes of different azimuthal order  $m$  are decoupled they can be treated separately. On a round-trip through the resonator the eigenmodes must be transmitted self-consistently at resonance. The characteristic equation can be formulated from this condition. For a given geometry of the resonator and a known material its solutions are the complex resonance frequencies  $f = f' + jf''$  of modes  $C_{mnq}$ . The subscripts denote the azimuthal ( $m$ ), the radial ( $n$ ), and the longitudinal ( $q$ ) order. Alternatively, for instance,  $f'$  may be set,  $l$  and  $f''$  being the unknowns [2]. This approach offers numerical advantages as the number of time-consuming mode expansions at the short-circuits is reduced by about 50%. However, it only works for length-dependent modes. An in-depth discussion of the results is found in [2] for modes with  $m = 1$ .

For completeness and in order to fully understand experimental findings it is necessary to explore the whole mode spectrum in the considered frequency range, including, in particular, the length-independent modes. As these are purely transversal  $k_z^+ = k_z^- = 0$  must be fulfilled. Here,  $k_z^+$  and  $k_z^-$  are the (longitudinal) propagation constants of the underlying left and right circulating waveguide modes. Interestingly, only modes with no azimuthal dependence, i.e. with  $m=0$  satisfy the above

condition. This becomes clear from a look at the dispersion characteristics of circular chirowaveguides [3]. Because of mode-splitting the conditions  $k_z^+ = 0$  and  $k_z^- = 0$  cannot be satisfied simultaneously, i.e. at the same frequency, except for the degenerate modes: These have no azimuthal dependence, that is  $m = 0$ , and are the only ones that give rise to length-independent resonator modes ( $q = 0$ ). The characteristic equation then reads:  $|k_z^+| = |k_z^-| = 0$ . It is solved for the complex frequency. The unloaded quality factor is given by  $Q \equiv f'/2f''$ .

Fig.1 depicts as a typical numerical result the dependence between the resonator length and the real part of the resonance frequency  $f'$  for different values of the chirality parameter  $\beta$  (Drude-Born-Fedorov notation). The permittivity and permeability were set to  $\epsilon_r = 1 - j0.001$  and  $\mu_r = 1 - j0.001$ , respectively. The radius  $a$  is 0.025 m. The figure displays  $C_{11q}$ -modes of different longitudinal order and shows how the mode coupling at the short-circuits increases with the chirality parameter when the next higher order waveguide mode ( $C_{12}$ ) becomes propagating [1]. The cutoff frequencies of the fundamental  $C_{11}$ - and the  $C_{12}$ -mode of the chirowaveguide are indicated for reference. Fig.2 shows the Q-factor of the  $C_{11q}$ -mode. It decreases with increasing chirality parameter and exhibits strong variations in the vicinity of the  $C_{12}$ -cutoff.

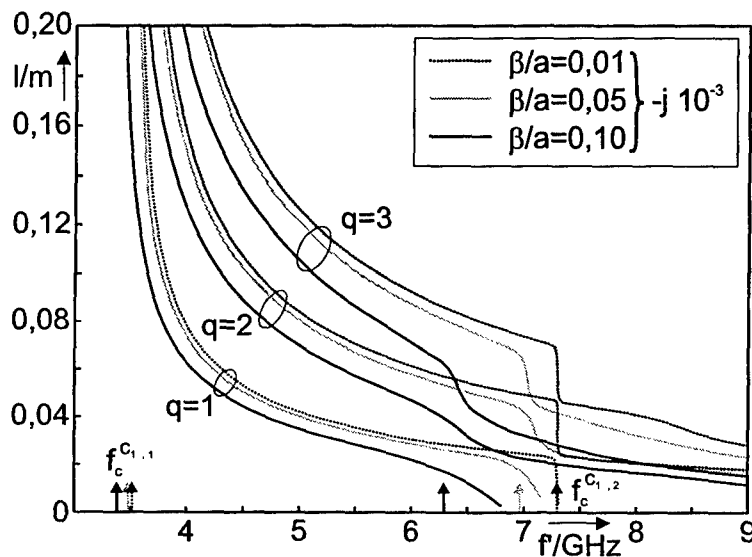


Fig. 1 Resonator length vs. frequency for different chiral parameters and longitudinal orders.

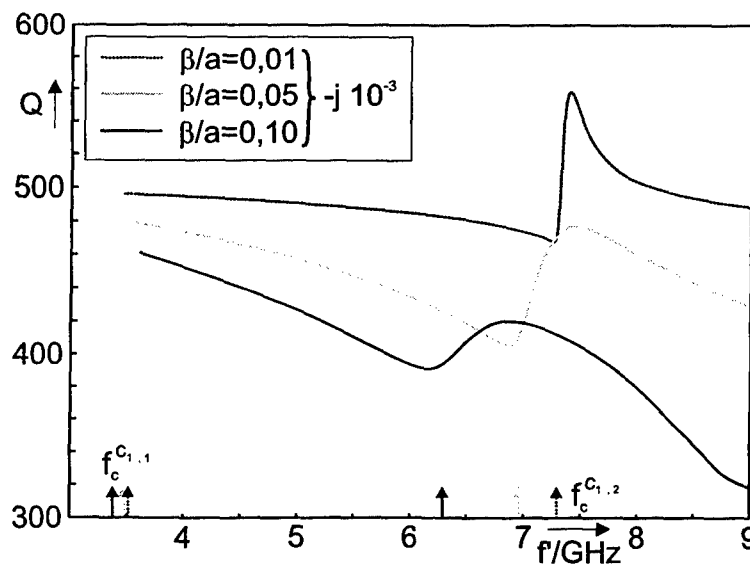


Fig. 2 Resonator quality factor  $Q$  for different values of the chirality parameter (longitudinal order  $q = 2$ )

### 3. Experimental Setup

The experimental setup is shown in Fig.3. It consists of a short-circuited piece of circular waveguide of radius  $a = 0.025$  m. To allow a verification over a broad frequency range rather than at only a few discrete frequencies one of the shorts is realized as movable plunger. The length can be extended up to  $l = 0.5$  m. The resonator is operated in transmission mode. To this end a microstrip antenna (22 mm long) is mounted radially on each short. This flat design allows to rotate the antennas azimuthally for optimum measurement sensitivity without damaging them or the chiral material.

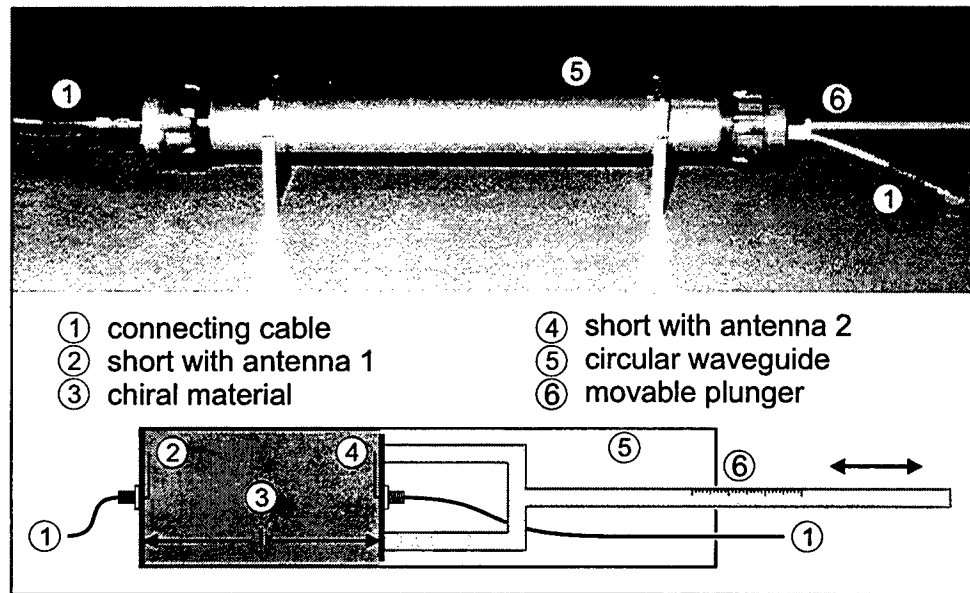


Fig. 3 Experimental setup.

### 4. Experimental Results

To validate the apparatus it was filled with a composite, non-chiral medium, i.e. spheres of PU-foam. The resonator was characterized with respect to its first four modes. Their resonance frequency is shown in Fig.4 versus the length of the resonator. The measured results correlate well with the values calculated on the basis of the dielectric constant ( $\epsilon_r = 1.11 - j0.003$ ) of the foam, that was obtained from an independent precision resonator measurement. In general, only slight variations can be observed. They are randomly distributed and are due to local changes of the density. They become reproducible only for the  $TM_{010}$ -mode at very short resonator length when the disturbance caused by the antennas becomes noticeable.

Next the resonator was filled with the chiral material of [4], and the resonator characteristics were recorded versus its length. The results are marked by crosses in Fig.4. They display a noticeable uncertainty which is attributable to the inhomogeneity of the material. Still, the general behavior is qualitatively in good agreement with the theoretical results presented above. The setup was then simulated with the model sketched above. The parameters of the (same) chiral material were taken from waveguide measurements as in [4]. Six different sets of data were used. When computing the resonance frequency of the  $C_{010}$ -mode one has to take into account the dispersion of the material parameters. The results are displayed as solid lines in Fig.4. The difference of the measured and the computed resonant frequencies remains within a few percent. Because of the comparably high sensitivity of the resonator measurement, this confirms the quality of the waveguide experiments. Due to larger measurement uncertainties the agreement is less pronounced in the cases where the samples used for the waveguide measurement had small differences in length [4]. The uncertainties observed in [4] when determining  $\epsilon_r$  and  $\mu_r$  of the chiral material do not appear, here. They were related to the

measurement errors on the ratio  $\mu_r/\epsilon_r$ , a quantity that does not affect the characteristic equation of the resonator.

The results obtained for the quality factor are less satisfactory. On the one hand, the values calculated from the measured material parameters decrease in average from 120 to 25 with the wavelength. On the other hand, the measured results exhibit a rather erratic behavior. They vary between 100 and 250 and show no significant frequency dependence. This is only to some extent attributable to the uncertain assessment of losses inherent to transmission measurements as in [4]. The measured  $Q$  appears to be much more sensitive to material inhomogeneities.

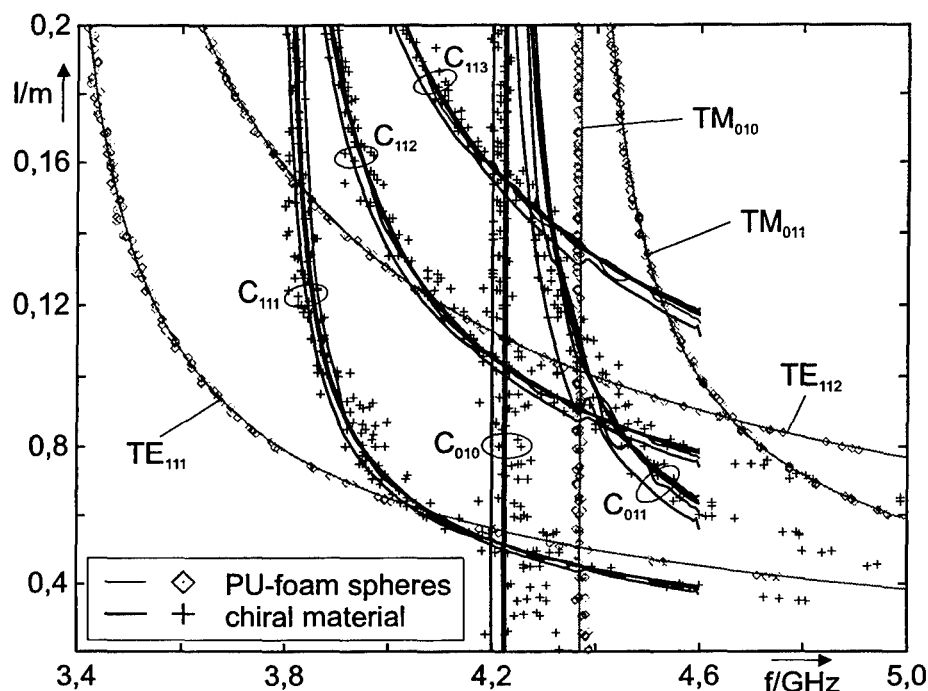


Fig. 4 Resonator length vs. measured and simulated resonance frequencies for PU-foam spheres and chiral material (modes of different order).

## 5. Conclusion

A resonator experiment was proposed to partially validate the constitutive parameters of a chiral material determined from waveguide measurements. To this end the resonator theory was extended to also include length-independent modes. These turn out to have no azimuthal dependence. The good correlation between the measured resonance frequencies and the ones calculated from the constitutive parameters confirms the quality of the waveguide experiments.

## References

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